Positive parity pentaquarks in a Goldstone boson exchange model

Fl. Stancu*

Université de Liège, Institut de Physique B.5, Sart Tilman, B-4000 Liège 1, Belgium

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Abstract

We study the stability of the pentaquarks $uudd\overline{Q}$, $uuds\overline{Q}$ and $udss\overline{Q}$ ($Q = c$ or b) of positive parity in a constituent quark model based on Goldstone boson exchange interaction between quarks. The pentaquark parity is the antiquark parity times that of a quark excited to a p-shell. We show that the Goldstone boson exchange interaction favors these pentaquarks much more than the negative parity ones of the same flavour content but all quarks in the ground state. We find that the nonstrange pentaquarks are stable against strong decays.

The existence of particles made of more than three quarks is an important issue of QCD inspired models. The interest has been raised so far by particles described by the colour state $[222]_C$, the tetraquarks $q^2\overline{q}^2$, the pentaquarks $q^4\overline{q}$ and the hexaquarks q^6. The present study is devoted to pentaquarks, first proposed independently by Gignoux, Silvestre-Brac and Richard [1] and Lipkin [2] about ten years ago. Within a constituent quark model based on one-gluon exchange (OGE) interaction, these authors found that the states $P^0_{\bar{c}s} = |uuds\bar{c}\rangle$ and $P^-_{\bar{c}s} = |uudds\bar{c}\rangle$ and their conjugates are stable against strong decays. Within better approximations, they turned out to be unstable [3]. A systematic theoretical study [4] in a model with OGE interaction suggested several candidates for stability, and especially those

*e-mail : fstancu@ulg.ac.be

with strangeness S = -1 or -2. In particular, the $uuds\bar{c}$ system was bound by -52 MeV. The nonstrange systems $uudd\overline{Q}$ (Q = c or b) were unbound.

If bound, the lifetime of the pentaquark $uuds\bar{c}$ or $udds\bar{c}$ is expected to be comparable to that of the $D_s^{\pm}$ meson [5] . The typically estimated pentaquark production cross section is of the order of 1 % that of $D_s^{\pm}$ [6]. Based on these predictions, experiments are being planned and the first search for the pentaquarks $P_{\bar{c}s}^0$ and $P_{\bar{c}s}^-$, performed at Fermilab, has just been reported [7]. No convincing evidence for strange pentaquarks was observed so far.

The theoretical predictions are definitely model-dependent. Reference [8], discussed the stability of heavy-flavoured pentaquarks within a chiral constituent quark model [9–11]. In this model, the hyperfine splitting in hadrons is obtained from the short-range part of the Goldstone boson exchange (GBE) interaction between quarks, instead of the OGE interaction of conventional models. The main merit of the GBE interaction is that it reproduces the correct ordering of positive and negative parity states in all parts of the considered spectrum, in contrast to any other OGE model. On the other hand, in its present form, it does not apply to hyperfine splitting in mesons. But the GBE interaction induces a strong short-range repulsion in the Λ-Λ system, which suggests that a deeply bound H-dibaryon should not exist [12]. This is in agreement with the high-sensitivity experiments at Brookhaven [13] where no evidence for H production was observed.

Reference [8] considered pentaquarks with strangeness ranging from S = -3 to S = 0. There, it was assumed that all light quarks are identical and the ground-state orbital (O) wave function is symmetric under permutation of light quarks, i.e. it corresponds to the Young diagram $[4]_O$. The subsystem of light quarks must necessarily be in a $[211]_C$ state. Then, the Pauli principle allows a certain number of spin $[f]_S$ and flavour $[f]_F$ states to be combined with $[4]_O$ and $[211]_C$ to give a total antisymmetric four-quark state. It was found that any of these states together with a heavy antiquark $\overline{Q}$ where Q = c or b gave rise to a pentaquark energy which was at least 300-400 MeV above the dissociation threshold nucleon plus meson, i.e. the considered pentaquarks cannot be bound compact objects. Their parity is P = -1, due to the antiquark.

The novelty of this study is that, within the same GBE model, we analyse the stability of pentaquarks with P = +1. In such a case, the parity of the pentaquark is given by P = $(-)^{\mathrm{L}+1}$, with L odd. Here we consider the case L = 1 and construct below the lowest variational solution where the light quarks carry an angular momentum L odd. This implies that the subsystem of four light quarks must be in a state of orbital symmetry $[31]_O$, where a light quark is excited to the p-shell. In this case the resulting intrinsic angular momentum is also the total angular momentum of the pentaquark system. Although the kinetic energy of such a state is higher than that of the $[4]_O$ state, a schematic estimate suggests that $[31]_O$ would lead to a stable pentaquark. Below we give the arguments of Ref. [14] based on a simplified GBE interaction of the form

$$V_\chi \;=\; -\,C_\chi \sum_{i\,<\,j} \lambda_i^F.\lambda_j^F \;\vec{\sigma}_i.\vec{\sigma}_j \tag{1}$$

with λ_i^F the Gell-Mann matrices, $\vec{\sigma}_i$ the Pauli matrices and $C_\chi \cong 30$ MeV, determined from the Δ-N splitting [9]. In the spirit of Ref. [9], there is no meson-exchange interaction between quarks and antiquarks. It is assumed that the $q\overline{q}$ pseudoscalar pair interaction is automatically included in the GBE interaction. Then, for the spin-spin interaction, the discussion is restricted to the light q^4 subsystem. The Pauli principle allows the following lowest totally antisymmetric states with $[31]_O$ symmetry

$$|1\rangle \;=\; \Big([31]_O[211]_C\Big[1^4\Big]_{OC}\;;\;[22]_F[22]_S[4]_{FS}\Big) \tag{2}$$

$$|2\rangle \;=\; \Big([31]_O[211]_C\Big[1^4\Big]_{OC}\;;\;[31]_F[31]_S[4]_{FS}\Big) \tag{3}$$

The expectation value of (1) calculated, for example, according to the Appendix of Ref. [15], is $-28\;C_\chi$ for $|1\rangle$ and $-64/3\;C_\chi$ for $|2\rangle$. These two states would actually couple via a quark-antiquark spin-spin interaction to a total angular momentum $J = 1/2$ or $3/2$ where $\vec{J} \;=\; \vec{L} + \vec{S} + \vec{s}_Q$, with $\vec{L}$, $\vec{S}$ the angular momentum and spin of the light system and $\vec{s}_Q$ the spin of the antiquark. As the quark-antiquark interaction is neglected here, in the following we restrict our discussion to the lowest state, i.e. $|1\rangle$. The quark-antiquark interaction is

neglected in the description of mesons as well, as for example in Ref. [16], so that the meson Hamiltonian contains a kinetic and a confinement term only.

We are interested in the quantity

$$\Delta E = \ E(q^4\overline{Q}) - \ E(q^3) - \ E(q\overline{Q}) \tag{4}$$

In our schematic estimate, we suppose that the confinement energy roughly cancels out in ΔE. Then, the kinetic energy contribution to ΔE is $\Delta KE = 5/4\ \hbar\omega$ in a harmonic oscillator model and the GBE interaction associated with the state $|1\rangle$ leads to $\Delta V_\chi = -14\ C_\chi$. With $\hbar\omega \approx 250$ Mev. determined from the N(1440) - N splitting [9], this gives

$$\Delta E \ = \ 5/4\ \hbar\omega \ - \ 14\ C_\chi \ = \ -\ 107.5\ MeV \tag{5}$$

i.e. a substantial binding. This is to be contrasted with the negative parity pentaquarks studied in Ref. [8] where the lowest state with S = 0 has the structure $|[4]_O[211]_C[211]_{OC};[211]_F[22]_S[31]_{FS}\rangle$. In this case the expectation value of (1) is $-16\ C_\chi$. Hence $\Delta E \ = \ 3/4\ \hbar\omega \ - \ 2\ C_\chi \ = \ 127.5\ MeV$, This suggests that the pentaquarks of negative parity are unstable, consistent with the detailed study made in [8].

The estimate (5) is a consequence of the flavour dependence of the GBE interaction. For a specific spin state $[f]_S$, a schematic OGE interaction of type $V_{c\ m} \ = \ -\ C_{c\ m} \sum \lambda_i^c.\lambda_j^c\ \vec{\sigma}_i.\vec{\sigma}_j$ does not make a distinction between $[4]_O$ and $[31]_O$ so that the $[31]_O$ state will appear higher than $[4]_O$ due to the kinetic energy. The GBE interaction overcomes the excess of kinetic contribution in $[31]_O$ and generates a lower expectation value for $[31]_O$ than for $[4]_O$.

The GBE Hamiltonian considered below has the form [10] :

$$H = \sum_i m_i + \sum_i \frac{\vec{p}_i^{\,2}}{2m_i} - \frac{(\sum_i \vec{p}_i)^2}{2\sum_i m_i} + \sum_{i<j} V_{\rm conf}(r_{ij}) + \sum_{i<j} V_\chi(r_{ij})\,, \tag{6}$$

with the linear confining interaction :

$$V_{\rm conf}(r_{ij}) = -\frac{3}{8}\lambda_i^c\cdot\lambda_j^c\, C\, r_{ij}\,, \tag{7}$$

and the spin–spin component of the GBE interaction in its $SU_F(3)$ form :

$$V_\chi(r_{ij}) = \left\{ \sum_{F=1}^{3} V_\pi(r_{ij}) \lambda_i^F \lambda_j^F \right.$$
$$\left. + \sum_{F=4}^{7} V_K(r_{ij}) \lambda_i^F \lambda_j^F + V_\eta(r_{ij}) \lambda_i^8 \lambda_j^8 + V_{\eta'}(r_{ij}) \lambda_i^0 \lambda_j^0 \right\} \vec{\sigma}_i \cdot \vec{\sigma}_j, \quad (8)$$

with $\lambda^0 = \sqrt{2/3}\,\mathbf{1}$, where $\mathbf{1}$ is the 3×3 unit matrix. The interaction (8) contains $\gamma = \pi, K, \eta$ and η' meson-exchange terms and the form of $V_\gamma\,(r_{ij})$ is given as the sum of two distinct contributions : a Yukawa-type potential containing the mass of the exchanged meson and a short-range contribution of opposite sign, the role of which is crucial in baryon spectroscopy. For a given meson γ, the exchange potential is

$$V_\gamma(r) = \frac{g_\gamma^2}{4\pi} \frac{1}{12 m_i m_j} \{\theta(r - r_0)\mu_\gamma^2 \frac{e^{-\mu_\gamma r}}{r} - \frac{4}{\sqrt{\pi}} \alpha^3 \exp(-\alpha^2 (r - r_0)^2)\} \quad (9)$$

For the Hamiltonian (6)-(9), we use the parameters of Refs. [10,12]. These are :

$$\frac{g_{\pi q}^2}{4\pi} = \frac{g_{\eta q}^2}{4\pi} = \frac{g_{Kq}^2}{4\pi} = 0.67,\ \frac{g_{\eta' q}^2}{4\pi} = 1.206,$$
$$r_0 = 0.43\,fm,\ \alpha = 2.91\,fm^{-1},\ \ C = 0.474\,fm^{-2},\ m_{u,d} = 340\,MeV,\ m_s = 440\,MeV, \quad (10)$$
$$\mu_\pi = 139\,MeV,\ \mu_\eta = 547\,MeV,\ \mu_{\eta'} = 958\,MeV,\ \mu_K = 495\,MeV.$$

The masses of the threshold hadrons are calculated variationally as in Ref. [8] where we assume an s^3 configuration for baryons. They are given in Table 1.

For pentaquarks, we used the internal Jacobi coordinates

$$\vec{x} = \vec{r}_1 - \vec{r}_2\,, \quad \vec{y} = (\vec{r}_1 + \vec{r}_2 - 2\vec{r}_3)/\sqrt{3}$$
$$\vec{z} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 - 3\vec{r}_4)/\sqrt{6}\,, \quad \vec{t} = (\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 - 4\vec{r}_5)/\sqrt{10} \quad (11)$$

First, we expressed the q^4 orbital wave functions of symmetry $[31]_O$ in terms of Jacobi coordinates. We assumed an s^3p structure for $[31]_O$ and inspired by Moshinski's method [17], we found the content of the three independent $[31]_O$ states [18], denoted below by ψ_i, in terms of shell model functions $|n\ \ell\ m\rangle$. The result is

$$\psi_1 = \begin{array}{|c|c|c|}\hline 1 & 2 & 3 \\ \hline 4 \\ \cline{1-1} \end{array} = \langle \vec{x}\,|000\rangle\,\langle \vec{y}\,|000\rangle\,\langle \vec{z}\,|010\rangle \quad (12)$$

$$\psi_2 = \begin{array}{|c|c|c|}\hline 1 & 2 & 4 \\ \hline 3 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|000\rangle\,\langle \vec{y}\,|010\rangle\,\langle \vec{z}\,|000\rangle \tag{13}$$

$$\psi_3 = \begin{array}{|c|c|c|}\hline 1 & 3 & 4 \\ \hline 2 \\ \cline{1-1}\end{array} = \langle \vec{x}\,|010\rangle\,\langle \vec{y}\,|000\rangle\,\langle \vec{z}\,|000\rangle \tag{14}$$

where, for convenience, we took the quantum number m = 0 everywhere. The pentaquark orbital wave functions ψ_i^5 are then obtained by multiplying each ψ_i by the wave function $\left\langle \vec{t}|000\right\rangle$ which describes the relative motion of the q^4 subsystem and the antiquark $\overline{Q}$. Assuming two variational parameters, a for the internal motion of q^4 and b for the relative motion of q^4 and $\overline{Q}$, we have explicitly

$$\psi_1^5 \;=\; N\;\exp\left[-\frac{a}{2}\left(x^2\;+\;y^2\;+\;z^2\right)\;-\;\frac{b}{2}\,t^2\right]\;z\;Y_{10}\;\;(\hat{z}) \tag{15}$$

$$\psi_2^5 \;=\; N\;\exp\left[-\frac{a}{2}\left(x^2\;+\;y^2\;+\;z^2\right)\;-\;\frac{b}{2}\,t^2\right]\;y\;Y_{10}\;\;(\hat{y}) \tag{16}$$

$$\psi_3^5 \;=\; N\;\exp\left[-\frac{a}{2}\left(x^2\;+\;y^2\;+\;z^2\right)\;-\;\frac{b}{2}\,t^2\right]\;x\;Y_{10}\;\;(\hat{x}) \tag{17}$$

where

$$N \;=\; \frac{2^{3/2}a^{11/4}b^{3/4}}{3^{1/2}\pi^{5/2}} \tag{18}$$

The kinetic energy part of (6) can be calculated analytically. For the state (2) or (3), its form is

$$\begin{aligned}\langle T\rangle \;&=\; \frac{1}{3}\,\left[\left\langle\psi_1^5\,|T|\,\psi_1^5\right\rangle\;+\;\left\langle\psi_2^5\,|T|\,\psi_2^5\right\rangle\;+\;\left\langle\psi_3^5\,|T|\,\psi_3^5\right\rangle\right] \\ &=\; \hbar^2\;\left(\frac{11}{2\mu_1}\;a\;+\;\frac{3}{2\mu_2}\;b\right)\end{aligned} \tag{19}$$

with

$$\frac{4}{\mu_1} \;=\; \begin{cases} \dfrac{1}{m_1}\;+\;\dfrac{3}{m_2} & for\;\;q_1q_2^3 \\[2ex] \dfrac{2}{m_1}\;+\;\dfrac{2}{m_2} & for\;\;q_1^2q_2^2 \end{cases} \tag{20}$$

where q_1, q_2 are light quarks and

$$\frac{5}{\mu_2} \;=\; \frac{1}{\mu_1} \;+\; \frac{4}{m_Q} \tag{21}$$

m_Q representing the heavy antiquark mass. Here, we choose $m_c = 1.35$ GeV and $m_b = 4.66$ GeV according to Ref. [8] . Taking $m_u = m_d = m_s = m_Q$ and setting a=b, the identical particle limit $\langle T\rangle \;=\; 7/2\;\hbar\omega$ with $\hbar\omega \;=\; 2\;a\hbar^2/m$ is recovered correctly.

Integrating in the colour space as shown in Ref. [8], the confinement part of (6) becomes

$$\langle V_{conf}\rangle \;=\; \frac{C}{2}\;\left(6\;\langle r_{12}\rangle \;+\; 4\;\langle r_{45}\rangle\right) \tag{22}$$

where the coefficients 6 and 4 account for the number of light-light and light-heavy pairs, respectively, and

$$\langle r_{ij}\rangle \;=\; \frac{1}{3}\;\left[\left\langle\psi_1^5\,|r_{ij}|\,\psi_1^5\right\rangle \;+\; \left\langle\psi_2^5\,|r_{ij}|\,\psi_2^5\right\rangle \;+\; \left\langle\psi_3^5\,|r_{ij}|\,\psi_3^5\right\rangle\right] \tag{23}$$

An analytic evaluation gives

$$\langle r_{12}\rangle \;=\; \frac{20}{9}\;\sqrt{\frac{1}{\pi a}} \tag{24}$$

and

$$\langle r_{45}\rangle \;=\; \frac{1}{3\sqrt{2\pi}}\;\left[2\sqrt{\frac{3}{a}\;+\;\frac{5}{b}}\;+\;\sqrt{5b}\;\left(\frac{1}{2a}\;+\;\frac{1}{b}\right)\right] \tag{25}$$

The matrix elements of the spin-flavour operators of (8) have been calculated using the fractional parentage technique described in Ref. [18] based on Clebsch-Gordan coefficients of the group S_4.

In Table 2, we present results for S = 0, S = -1 and S = -2 pentaquarks. The quantity ΔE, defined by (4), is obtained from $E\left(q^4\overline{Q}\right)$ calculated variationally for the state $|1\rangle$ defined by (2). The optimal values of the parameters a and b are indicated in each case. In all cases, one has $a \;>\; b$. The inverses 1/a and 1/b give an idea of the quark-quark and quark-antiquark distances, respectively. More precisely, these distances are proportional to the corresponding harmonic oscillator parameters. Due to the normalization (11) of the

Jacobi coordinates, taken in fermi units these parameters are $\hbar\omega/\sqrt{2a}$ and $\hbar\omega/\sqrt{2b}$. Then, using Table II, one gets for example, for $uudd\bar{c}$, 0.42 fm and 0.70 fm respectively. These estimates suggest that, at equilibrium, the light quarks are clustered together, orbitting around the heavy antiquark.

The present variational solution does not give binding for $uuds\overline{Q}$ and $udss\overline{Q}$. But the nonstrange positive parity pentaquarks $uudd\overline{Q}$ are bound by -75.6 MeV and -95.6 MeV for $Q = c$ and b, respectively. The reason is that the GBE interaction is stronger in the nonstrange case than in the strange one because of the $1/m_i m_j$ factor in Eq. (9). Thus, the GBE model suggests that the nonstrange positive parity pentaquarks are the best candidates for stable compact systems. This in contrast to OGE based models where strangeness is required [4] in order to reach stability for heavy-flavoured pentaquarks. Note that nonstrange pentaquarks associated with orbital symmetry $[4]_O$ and having total angular momentum $J = 1/2$ or $3/2$ are forbidden by Pauli principle (see ref. [8] Table 1). Hence strangeness is a necessary condition imposed by the color-spin structure of OGE operators to lowest pentaquark states.

The present results have important similarities with those obtained in [19] from the Skyrme model : 1) the lowest pentaquark states have positive parity for any flavour content and 2) stability does not necessarily require strangeness.

For nonstrange positive parity pentaquarks, the masses obtained in these calculations are $M\left(uudd\bar{c}\right) = 2.895$ GeV and $M\left(uudd\bar{b}\right) = 6.176$ GeV. One should be aware that these are upper bounds. Note however that their difference is very close to m_c - m_b, consistent with heavy quark effective theory [20].

In conclusion we encourage experimental search for nonstrange positive parity pentaquarks.

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REFERENCES

[1] C. Gignoux, B. Silvestre-Brac and J.-M. Richard, Phys.Lett. B193, 323 (1987).

[2] H.J. Lipkin, Phys. Lett. B195, 484 (1987).

[3] S. Fleck, C. Gignoux, J.-M. Richard and B. Silvestre-Brac, Phys.Lett. B220, 616 (1989).

[4] J. Leandri and B. Silvestre-Brac, Phys.Rev. D40, 2340 (1989).

[5] H.J. Lipkin, Nucl. Phys. A625, 207 (1997).

[6] M.A. Moinester, D. Ashery, L.G. Landsberg and H.J. Lipkin, Z.Phys. A356, 207 (1996).

[7] E.M. Aitala et al., Fermilab E791 Collaboration, Phys. Rev. Lett. 81, 44 (1998).

[8] M. Genovese, J.-M. Richard, Fl. Stancu and S. Pepin, Phys. Lett. B425, 171 (1998).

[9] L.Ya. Glozman and D.O. Riska, Phys. Rep. 268, 263 (1996); for earlier work see D. Robson, Topical Conference on Nuclear Chromodynamics, eds. J. Qiu and D. Sivers, World Scientific, Singapore (1988) p. 174.

[10] L.Ya. Glozman, Z. Papp and W. Plessas, Phys. Lett. B381, 311 (1996).

[11] L.Ya. Glozman, Z. Papp, W. Plessas, K. Varga and R.F. Wagenbrunn, Nucl. Phys. A623, 90c (1997).

[12] Fl. Stancu, S. Pepin and L.Ya. Glozman, Phys. Rev. D57, 4393 (1998).

[13] R. W. Stotzer et al., Phys. Rev. Lett. 78, 3646 (1997).

[14] L.Ya. Glozman and D.O. Riska, private communication.

[15] Fl. Stancu, S. Pepin and L.Ya. Glozman, Phys.Rev. C56, 2779 (1997).

[16] S. Pepin, Fl. Stancu, M. Genovese and J.-M. Richard, Phys. Lett. B393, 119 (1997).

[17] M. Moshinski, The Harmonic Oscillator in Modern Physics : from Atoms to Quarks, Gordon and Breach Science Publishers, New York 1969, Chapter 4.

[18] Fl. Stancu, Group Theory in Subnuclear Physics, Clarendon Press, Oxford 1996, Chapters 4 and 10.

[19] D.O. Riska and N.N. Scoccola, Phys. Lett. B299, 338 (1993)

[20] N. Isgur and M.B. Wise, Phys. Lett. B232, 113 (1989).

[21] Particle Data Group, Phys.Rev. D54, 1 (1996).

TABLES

TABLE I. Masses of hadrons required to calculate the threshold energy $E_T = E_{baryon} + E_{meson}$. The experimental mass for mesons represents the average $\overline{M} = (M + 3M^*)/4$, where M and M^* are the pseudoscalar and vector meson masses respectively [21] .

Hadron	Mass (GeV)	
	variational	experiment
N	0.970	0.939
Σ	1.235	1.192
$\overline{D}$	2.008	1.973
$\overline{D}_s$	2.087	2.076
$\overline{B}$	5.302	5.313
$\overline{B}_s$	5.379	5.375

TABLE II. Lowest positive parity pentaquarks of total angular momentum $J = 1/2, 3/2$. Column 1 gives the flavour content, column 2 the isospin I, columns 3 and 4 the optimal variational parameters of (15)-(18), column 5 gives $\Delta E = E - E_T$ where E is the expectation value of the Hamiltonian (6)-(10) and $E_T = E_{baryon} + E_{meson}$ for the threshold from the last column.

Pentaquark	I	Variational parameters (GeV^2)		ΔE	Threshold
		a	b	(MeV)	
$uudd\bar{c}$	0	0.110	0.040	-75.6	$N + \overline{D}$
$uudd\bar{b}$	0	0.110	0.053	-95.6	$N + \overline{B}$
$uuds\bar{c}$	1/2	0.101	0.041	104.7	$N + \overline{D}_s$
$uuds\bar{b}$	1/2	0.102	0.054	93.4	$N + \overline{B}_s$
$udss\bar{c}$	1	0.092	0.041	81.0	$\Sigma + \overline{D}_s$
$udss\bar{b}$	1	0.092	0.055	69.3	$\Sigma + \overline{B}_s$